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Bouguer density determination by fractal analysis

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ABSTRACT

Density values for the Bouguer reduction of two gravity data sets from Iceland are determined using a new method based on minimization of the roughness of the Bouguer anomaly surface. The fractal dimension of the surface is used as a gauge of the roughness. The analysis shows the size of topographic features supported by crust without isostatic compensation to be 25 to 30 km in southwest Iceland and 9 to 10 km inside the active rifting zone. The densities selected for these areas are 2490 and 2730 kg/m^3, respectively.

INTRODUCTION

In order to calculate the Bouguer gravity anomaly the average density (Bouguer density) for the topography whose gravitational influence is to be removed must first be computed. A common approach is to estimate this density by minimizing the resulting correlation of the Bouguer gravity anomaly with the topography (Nettleton, 1939) or other similar but more easily calculated quantities (Parasnis, 1962). The underlying assumption of these methods is that the topography is supported by a rigid crust rather than by isostatic compensation.

This study involves gravity data from an area in southwest Iceland well over 100 km in diameter. The area is suspected to be largely in isostatic equilibrium as the crustal thickness in the area is thought to range from 9 to 14 km (Palmason, 1971; Gebrande et al., 1980). This makes doubtful the minimizing of correlation between topography and Bouguer gravity, since the larger topographic features (which are not crustally supported) must be mirrored in the Bouguer gravity anomaly.

Using erroneously high or low Bouguer density values for the terrain correction leaves an excessive impression of the topography on the resulting Bouguer anomaly. Assuming the gravity field to be generally less rough than the topography, we determine the Bouguer density by minimizing the roughness of the resulting Bouguer anomaly. This roughness is estimated by the fractal dimension of the surface, which may be thought of as a measure of its chaotic roughness. A necessary condition is that the surface must be fractal, which is to say it maintains its apparent roughness independent of the scale at which we look at it.

The gravity data sets

The gravity data in the area of consideration fall into two distinct sets having two different scales of coverage (Figure 1). The more extensive data set, which covers the southwestern part

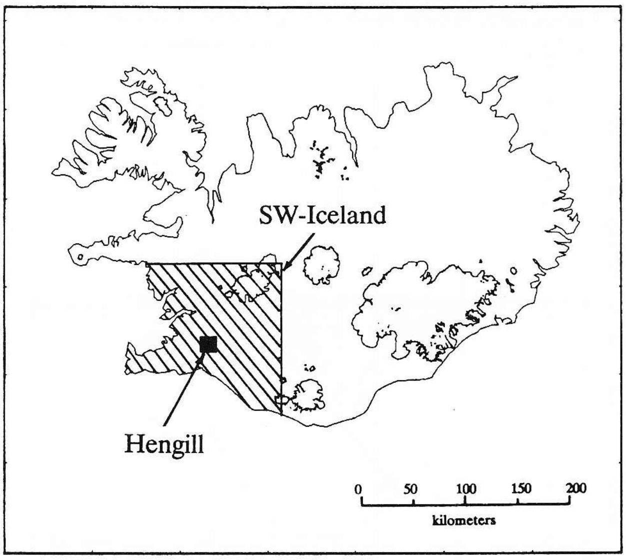

FIG. 1. The location of the two gravity data sets.

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of Iceland, consists of 291 measurement stations with an average distance between stations of about 8 km. The other data set covers a portion of the active volcanic zone thought to be a part of the mid-Atlantic ridge system. Within the area there is the central volcano Hengill from which the area derives its name, and a portion of a rift valley. The data consist of 313 stations with an average distance between stations of slightly less than one kilometer. A geological interpretation of these data has previously been offered (Thorarinsson et al., 1988).

Determination of the fractal dimension

Mandelbrot (1967) showed that some geographical lines such as coastlines, are best described in terms of a statistical property, which he termed the fractal dimension of the line. Fractal lines are self-similar, or scale-independent. The dimension D of a fractal line lies between 1 (the dimension of a straight line) and 2 (the dimension of a plane). Similarly surfaces can also have properties of self-similarity, or more exactly, self-affinity, i.e., self-similarity with vertical exaggeration. For such surfaces, known as fractional Brownian surfaces (Mandelbrot, 1975), the fractal dimensions will be between 2 (a plane) and 3 (a solid volume).

In this study the self-similarity and fractal dimension of gravitational surfaces are estimated using a method described by Mark and Aronson (1984), based on the variogram of a surface. To quote: "For a fractional Brownian surface of dimension $2 < D < 3$, the expected value of the squared elevation difference is given by

$$E\,[\,(Z_p - Z_q)^2\,] = k(d_{pq})^{2H}, \quad (1)$$

where Z_p and Z_q are the values of the surface at points p and q, d_{pq} is the horizontal distance between points, and H equals $3 - D$."

Plotting the variance of surface relief differences versus distance between points yields a graph on which a linear relationship over some range indicates self-similarity over that range. An estimate of the fractal dimension for that range is obtained from the slope b of a line through the points:

$$D = 3 - (\,b\,/\,2\,). \quad (2)$$

The calculations for a given data set are carried out as follows. The maximum distance between points is limited to the diameter of the largest circle that can be fitted within the area under consideration. This maximum distance is divided into 50 equal classes. The variance of surface differences is calculated for each class and plotted logarithmically versus the logarithm of the distance value of the class. Only classes with a minimum of 32 anomaly differences are plotted. The plot is then analyzed visually to determine whether a least squares regression line can be fitted to the values, or to some range of the values. Finally, the fractal dimension is derived from the slope of the fitted line.

The free air anomaly

The variograms of the free air anomaly maps show a definite linear relationship for short distances, but this relationship disappears at larger distances (Figures 2 and 3). The data from the Hengill area are fractal out to a range of 9 to 10 km while the other data set is fractal out to 25 to 30 km.

At short distances the free air anomaly correlates strongly with the local topography, which is supported by the lithosphere without deflection. The free air anomaly should then display fractal properties, as does the topography. However, as the size of the topographic anomalies increases, the lithosphere is deflected and the anomalous mass must be supported by isostatic compen-

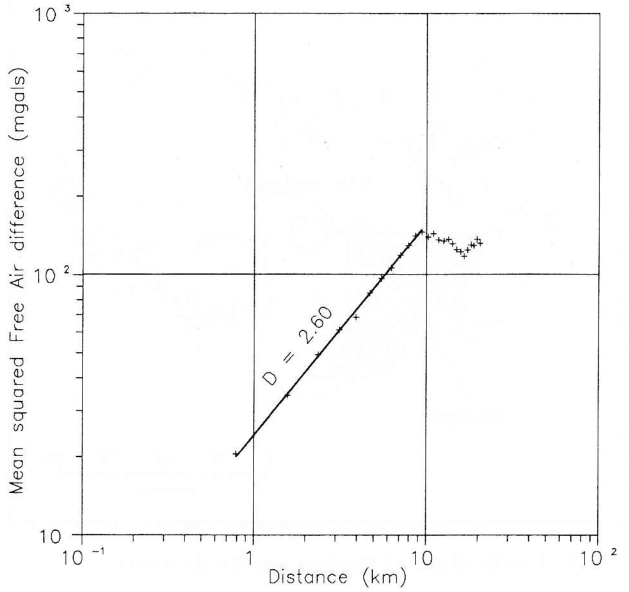

FIG. 2. Variogram for the free air anomaly in the Hengill area. The linear relationship out to 9 to 10 km indicates a fractal surface and is interpreted as crustally supported topography. The nonfractal character at larger distances is interpreted as isostatically compensated topography.

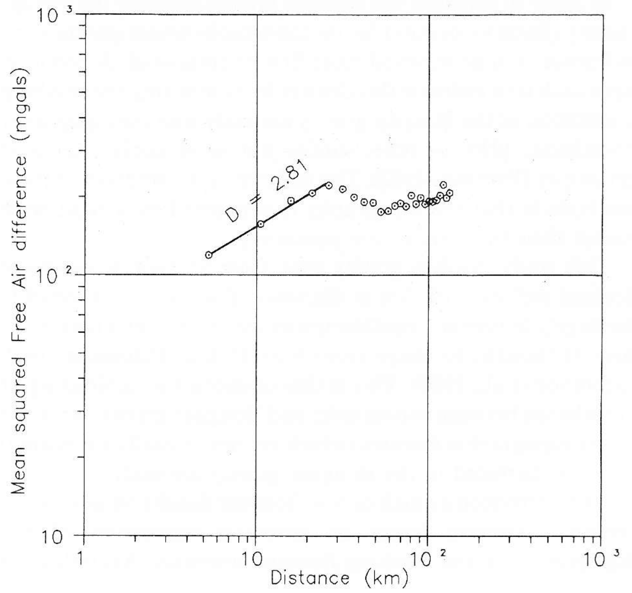

FIG. 3. Variogram for the free air anomaly in southwest Iceland. The linear relationship out to 25 to 30 km indicates a fractal surface and is interpreted as crustally supported topography. The non-fractal character at larger distances is interpreted as isostatically compensated topography.

sation. At these distances the free air anomaly should appear stochastic with a zero slope on the variogram.

Therefore we interpret the breaks in the plots as showing the range at which topographic anomalies stop being crustally supported and start being supported by isostatic compensation. The occurrence of the break points at different distances in the two plots is consistent with the fact that the thickness of the crust in the Hengill area, an active rifting zone, is smaller than the average crustal thickness of the entire southwestern part of Iceland.

This leads us to conclude that for both data sets the assumption of crustally supported topography is seriously wrong, and a determination of Bouguer density based on that assumption is tenuous at best. Since the larger topographic anomalies in both data sets are isostatically compensated, i.e. floating, they should be reflected in the Bouguer anomaly. Hence, the best density value for topographic corrections is not produced by minimizing the correlation between the topography and the Bouguer anomaly.

Determining the Bouguer density

The stated purpose of the Bouguer gravity correction is to eliminate the effect of the mass associated with the topography. In order to carry out the calculations we must derive an estimate of the density of the topographic mass. The leading part of the variogram presumably represents crustally supported topography. Thus, its fractal dimension may be used to arrive at a Bouguer density estimate. This is done by finding the Bouguer density that minimizes the fractal dimension of the leading end of the variogram for the Bouguer anomaly.

In the Hengill data the fractal dimension from 2.36 km out to the distance class of 9.45 km was minimized. This leaves the first two class values out of the variogram, as they appear nonfractal after the Bouguer correction (Figure 4). In the case of the southwest Iceland data, all class values out to 26.34 km were included in the fractal estimation (Figure 5).

Figure 6 shows the plot of the resulting fractal dimensions versus the densities. The analysis of the Hengill data reveals an average density of 2730 kg/m^3, while the entire southwestern part of Iceland has an average density of 2490 kg/m^3. These are the Bouguer densities used to compute the variograms shown in Figures 4 and 5.

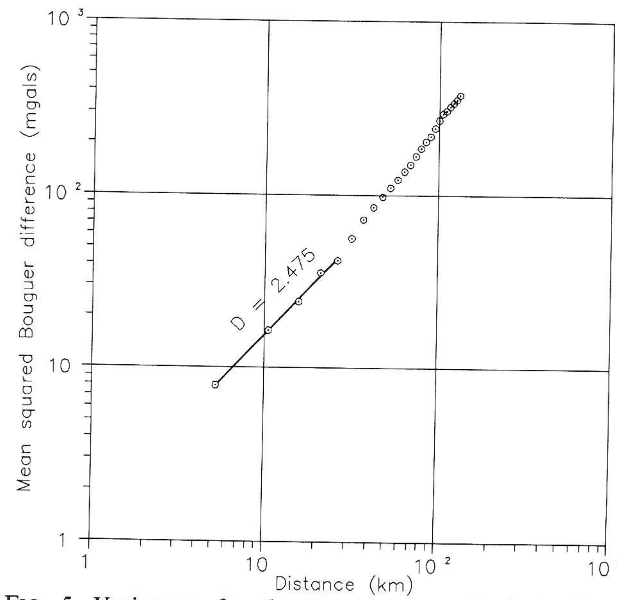

FIG. 5. Variogram for the Bouguer anomaly in southwest Iceland indicating two different fractal dimensions for the data. Between 5 and 30 km the plot reflects crustally supported topography with a high fractal dimension, but beyond that the crust starts to float, resulting in a lower fractal dimension for the Bouguer anomaly.

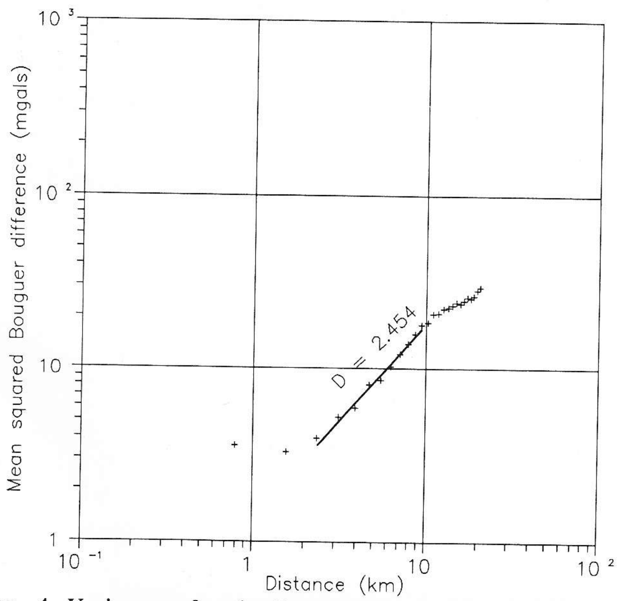

FIG. 4. Variogram for the Bouguer anomaly in the Hengill area; linear for distances between approximately 2 and 10 km, but nonfractal outside that range.

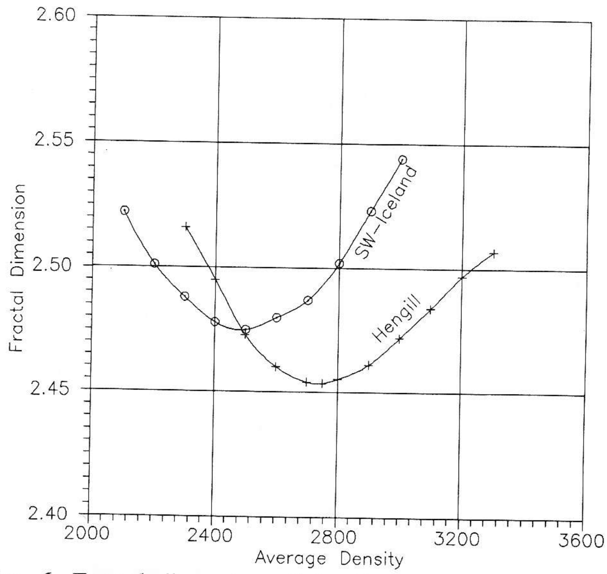

FIG. 6. Fractal dimension of the Bouguer anomaly in the crustally supported range as a function of densities used in the topographic correction.

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